10 Geometry, angles

1 Basic terms

Points

A point is one of the basic terms in geometry. We say that a point is a "dot" on a piece of paper. We identify this point with a number or letter. A point has no length or width.

Lines

A line is a "straight" line that we draw with a ruler on a piece of paper; a line extends forever in both directions.

Rays

A ray is a "straight" line that begins at a certain point and extends forever in one direction. The point where the ray begins is known as its endpoint.

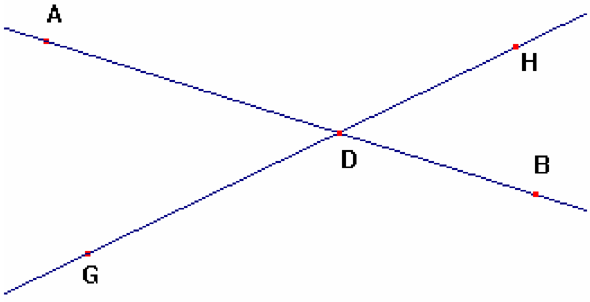
Line Segments

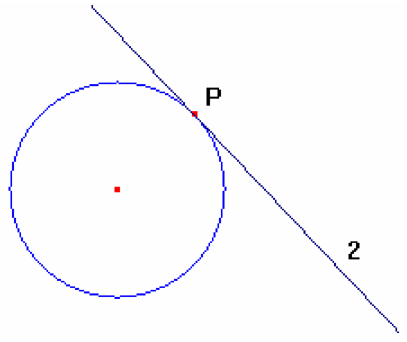
A line segment is a portion of a "straight" line. A line segment does not extend forever, but has two distinct endpoints. We write the name of a line segment with endpoints A and B as $\overline{AB}$.

Intersection

The term intersect is used when lines, ray lines or segments share a common point. The point they share is called the point of intersection.

Example: In the diagram below, line AB and line GH intersect at point D; line 2 intersects the circle at point P:

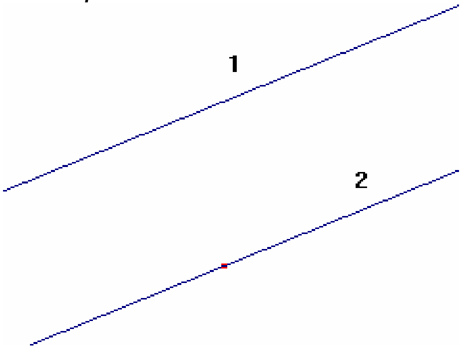




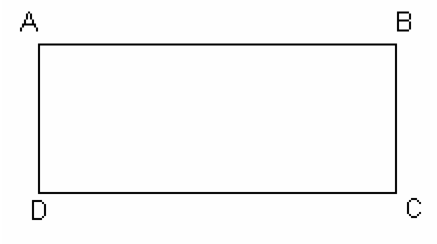
Parallel Lines

Two lines in the same plane which never intersect are called parallel lines. We say that two line segments are parallel if the lines that they lie on are parallel.

Examples: Lines 1 and 2 below are parallel.

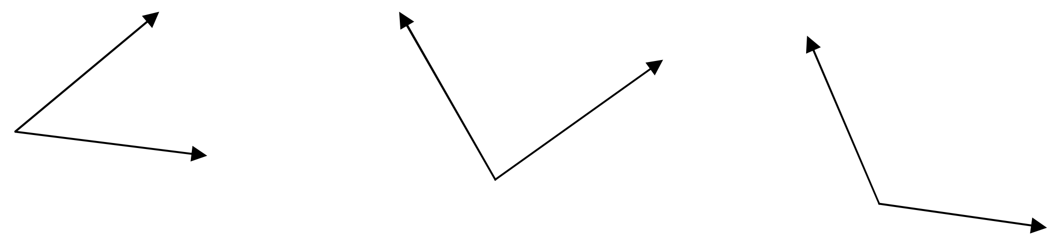


The *opposite* sides AB, DC or AD, BC of the rectangle below are parallel.



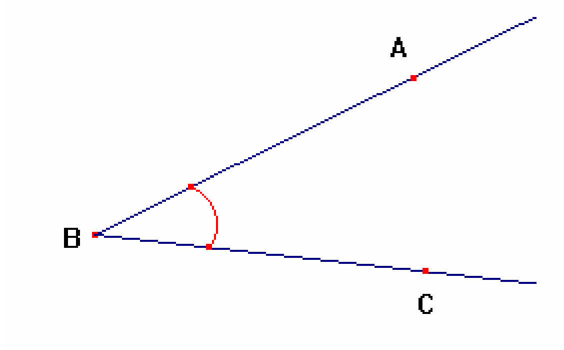
2 Angles

Two rays with the same endpoint form an angle. The point where the rays intersect is called the vertex of the angle. The two rays are called the sides of the angle.
Some examples of angles are:



2.1 How to name the angles.

We can name the angle below as $\angle B$ or even **b,** but it is better to name it as $\angle$ABC or as $\angle$CBA. Note how the vertex point is always given in the middle.

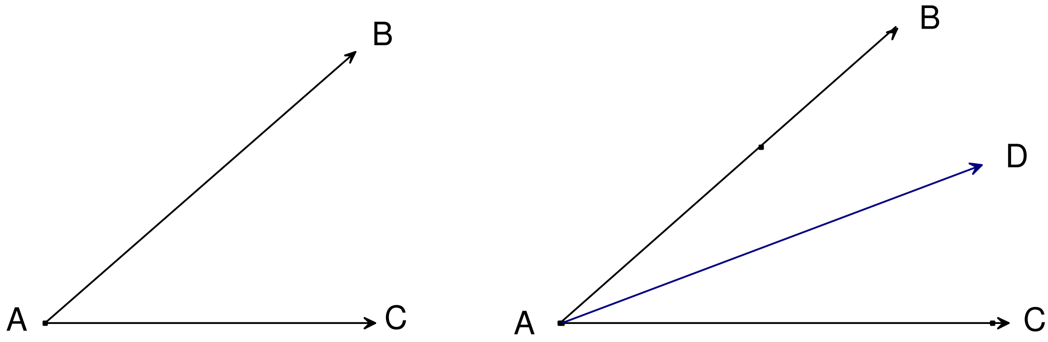


2.2 Angle Bisector

An angle bisector is a ray that divides an angle into two equal angles.

Example

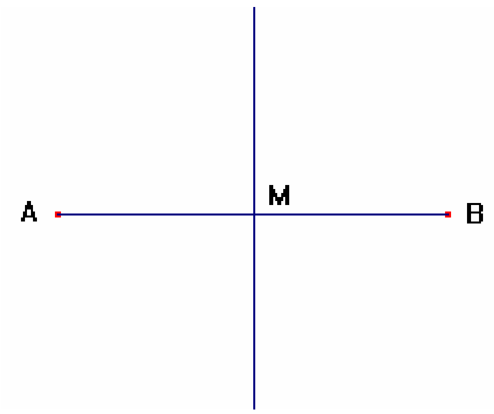
The ray AD on the right is the angle bisector of the angle BAC on the left because $\angle BAC = 2\cdot \angle BAD = 2\cdot \angle DAC$.



2.3 Bisecting line of a segment

Two lines that meet at a right angle are perpendicular.

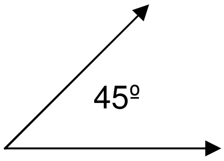
The perpendicular line that divides a segment into two equal parts is the bisecting line of the segment

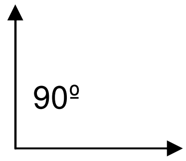


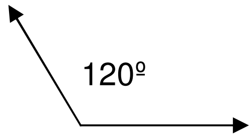
2.4 Measuring angles. Degrees

We measure the size of an angle using degrees.
There are 360º in a full turn.

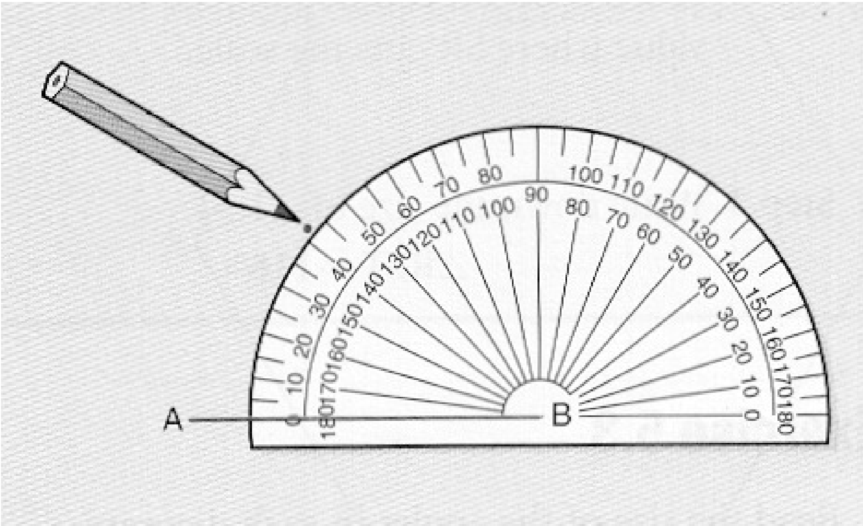
Here are some examples of angles and their approximate degree measurements.







For drawing and measuring angles we use a **protractor**.



Note that usually there are two scales in the protractors, use the correct one.

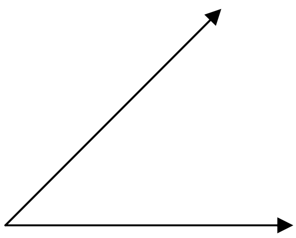
Exercise 1

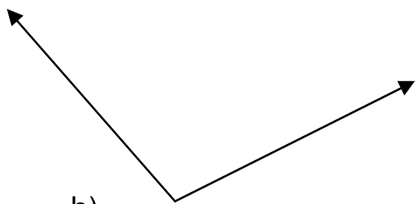
1.1 Draw in your notebook the following angles:

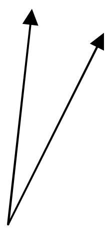
a) ∠ABC = 23º b) ∠CDE = 56º c) ∠PQR = 88º

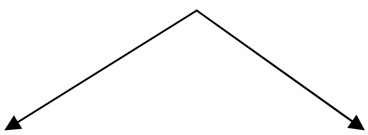
d) ∠ RST = 68º e) ∠KRT = 144º f) ∠MVL = 180º

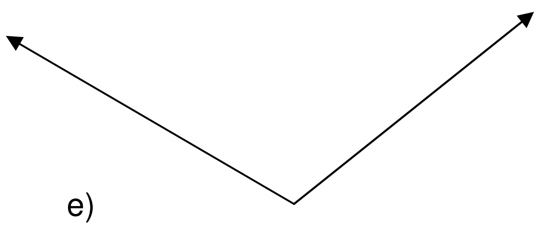
1.2 Measure the size of the following angles:

a) 

b) 

c) 

d) 

e) 

Approximations

- 1° is approximately the width of a pinky finger nail at arm's length
- 10° is approximately the width of a closed fist at arm's length.

3 Type of angles

Acute angles: An acute angle is an angle measuring between 0 and 90 degrees.

Obtuse angles: An obtuse angle is an angle measuring between 90 and 180 degrees.

Right angles: A right angle is an angle measuring 90 degrees. Two lines or line segments that meet at a right angle are perpendicular.

Straight angles: A straight angle is an angle measuring 180º

Complete turn: One complete turn is 360º

Exercise 2

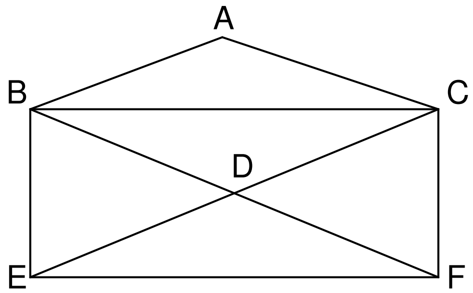
a) Make a list of objects in the classroom or in your house which contain:

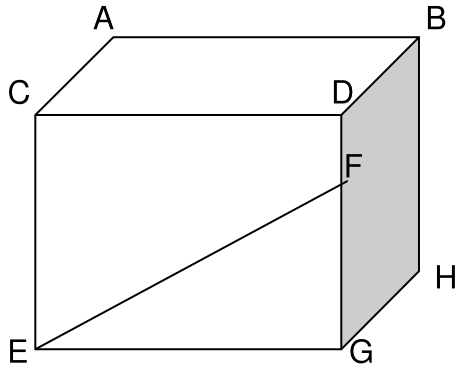
A right angle

A straight angle

An acute angle

b) Name all the obtuse, acute and right angles in the diagrams.





Exercise 3

How many degrees will the minute hand of a clock move in

a) 15 minutes **b) 30 minutes**

c) 5 minutes **d) 20 minutes**

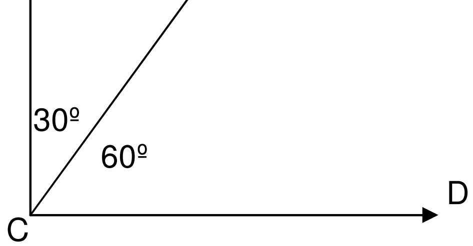
e) 1 minute **f) 46 minutes**

4 Related angles

Complementary Angles

Two angles are called complementary angles if the sum of their degree measurements equals 90 degrees. One of the complementary angles is the **complement** of the other.

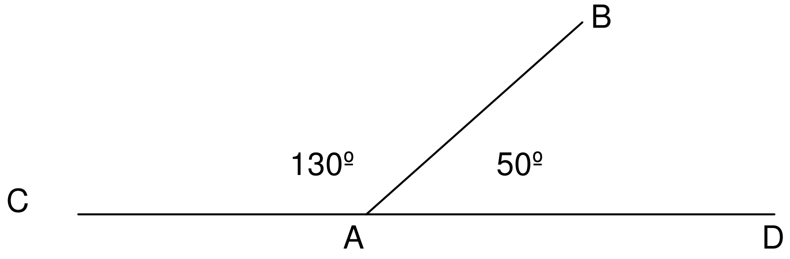
The two angles $\angle DCB = 60º$ and $\angle ACB = 30º$ are complementary.



Supplementary Angles

Two angles are called supplementary angles if the sum of their degree measurements equals 180 degrees. One of the supplementary angles is the **supplement** of the other.

The two angles $\angle DAB = 50º$ and $\angle CAB = 130º$ are supplementary.



Exercise 4 Calculate the complement and draw them on your notebook :

a) 17º

b) 42º

c) 7º

d) 90º

Exercise 5 Calculate the supplement of:

a) 37º

b) 52º

c) 123º

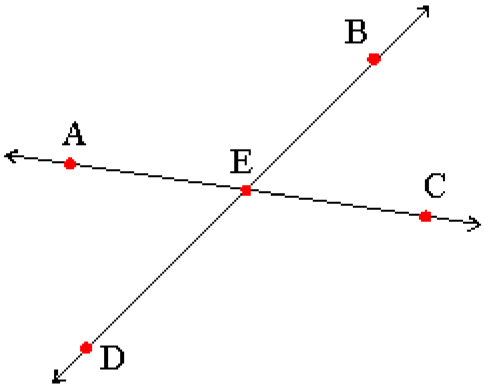
d) 7º

e) 90º

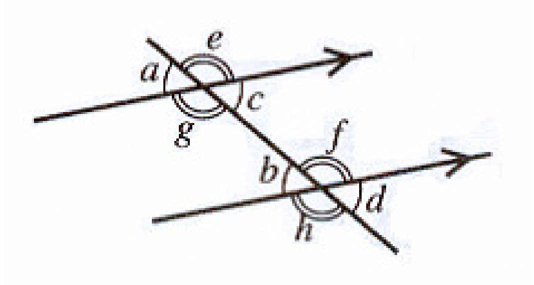
5 Angles between intersecting lines

Vertical Angles

For any two lines that meet, such as in the diagram below, angle $\angle BEC$ and angle $\angle AED$ are called vertical angles or vertical opposite angles. Vertical angles have the same degree measurement. Angle $\angle AEB$ and angle $\angle DEC$ are also vertical angles.



When a line crosses two parallel lines like in the diagram below



a = b = c = d

And

e = f = g = h

c and b are called **Alternate Interior Angles**

g and f are also **Alternate Interior Angles**

e and h are called **Alternate Exterior Angles**

a and d are also **Alternate Exterior Angles**

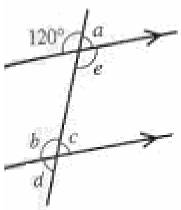
e and f are called **Corresponding Angles**

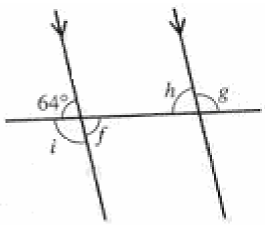
a and b are also **Corresponding Angles**

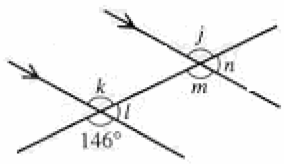
Remember that f and h or b and d, for example, are **Vertical Angles**

Exercise 6

a) Find the unknown angles and say all the correspondences you can see:





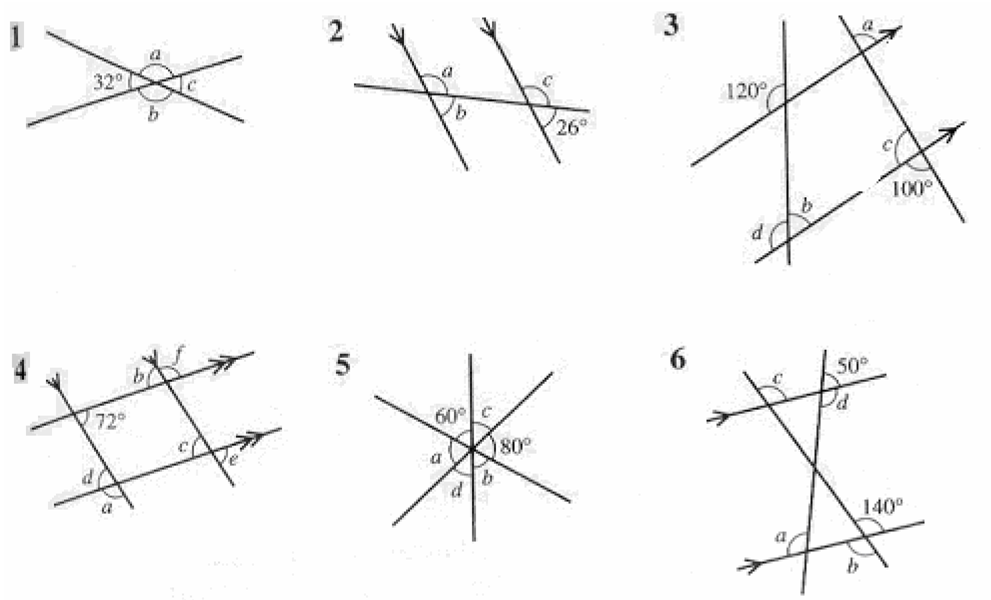


a = **b =** **c =** **d =** **e =** **f =**

g = **h =** **i =** **j =** **k =** **l =**

m = **n =**

b) Find the unknown angles:



1. a = **b =** **c =**

2. a = **b =** **c =**

3. a = **b =** **c =** **d =**

4. a = **b =** **c =** **d =** **e =** **f =**

5. a = **b =** **c =** **d =**

6. a = **b =** **c =** **d =**

6 Operations with angles

The subunits of the degree are the **minute** of arc and the **second** of arc.

One minute $1' = \frac{1}{60}$ of a degree, that is $1^{o} = 60'$

One second $1'' = \frac{1}{60}$ of a minute, that is $1' = 60''$

These are the sexagesimal units, so an angle "a" can be expressed for example $a = 25^{o}\,23'\,40''$ and we need to operate angles expressed in this form.

6.1 Addition.

We need to add separately degrees minutes and seconds and then convert the seconds into minutes and the minutes into degrees if we get more than 60 subunits.

Example

Add $45^{o}\,53'\,40'' + 12^{o}\,33'\,35''$

Adding separately we get $45^{o}\,53'\,40'' + 12^{o}\,33'\,35'' = 57^{o}\,86'\,75''$ but $75'' = 1'\,15''$ so we add $1'$ and get $87' = 1^{o}\,27'$ we add 1^{o} and finally the solution is $57^{o}\,86'\,75'' = 58^{o}\,27'\,15''$

6.2 Subtraction

We need to subtract separately degrees minutes and seconds, if we do not have enough seconds or minutes we convert one degree into minutes or a minute into seconds.

Example

Subtract $57^{o}\,13'\,21''$ and $12^{o}\,43'\,35''$ We write $57^{o}\,13'\,21''$ as $56^{o}\,73'\,21''$ and then:

$$\begin{array}{r} 56^{o}\,72'\,81'' \\ -\;12^{o}\,43'\,35'' \\ \hline 44^{o}\,29'\,46'' \end{array}$$

Is the answer

6.3 Multiplication by a whole number

We multiply separately degrees minutes and seconds and then convert the seconds into minutes and the minutes into degrees when we get more than 60 subunits.

Example

Multiply $(22^{\circ}\,13'\,25'')\cdot 6$

22º	13'	25"
6	6	6
132	78	150
	80'	2' 30"
133º	1º 20'	

Solution 133º 20' 30"

6.4 Division by a whole number

We divide the degrees, and the remainder is converted into minutes that must be added to the previous, divide the minutes and we repeat the same that we have done before. The remainder is in seconds.

Example

Divide $(22^{\circ}\,13'\,25''):4$

22º	13'	25"	4
2º x 60 =	120		5º 33' 21" quotient
	133'		
	1' x 60 =	60"	
		85"	
		1" remainder	

Exercise 7

7.1 Add:

a) 28º 35' 43" + 157º 54' 21"

b) 49º 55' 17" + 11º 5' 47"

c) 233º 5’ 59” + 79º 48’ 40”

7.2

a) Subtract 34º 32’ 12” – 11º 30’ 22”

b) Calculate the complement of 13º 45’ 12”

c) Calculate the supplement of 93º 30’

7.3 Given A = 22º 32’ 41” Calculate:

a) 2·A

b) 3·A

c) $\frac{A}{5}$

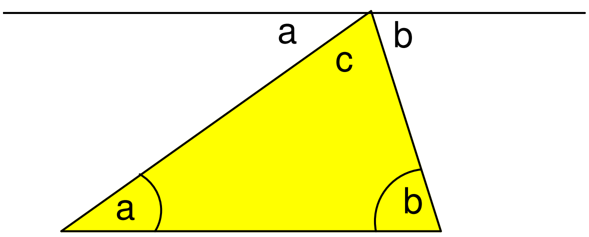
d) $\frac{2 \cdot A}{5}$

7 Angles in the polygons

7.1 Triangle

A triangle is a three-sided polygon.

The sum of the angles of a triangle is 180 degrees.

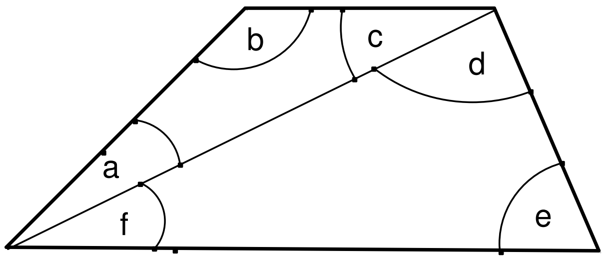


$a + b + c = 180º$

7.2 Quadrilateral

A quadrilateral is a four-sided polygon

The sum of the angles of any quadrilateral is 360º



$a + b + c = 180º \quad d + e + f = 180º$

$a + b + c + d + f = 360º$

7.3 Polygon of n sides

The sum of the angles of a polygon with *n* sides, where *n* is 3 or more, is $(n-2)\cdot 180º$.

7.4 Regular Polygon

A regular polygon is a polygon whose sides are all the same length, and whose angles are all the same.

As the sum of the angles of a polygon with *n* sides is $(n-2)\cdot 180º$, each angle in a regular polygon is $\frac{(n-2)180º}{n}$.

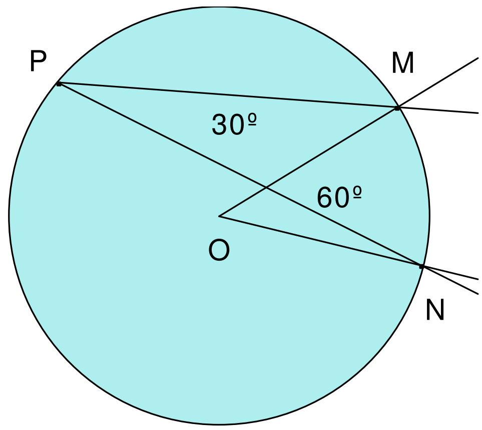
Exercise 8 Complete the following table

Name	Number of sides	Sum of the interior angles	Name of the regular polygon	Interior angle
Triangle	3	180º	Equilateral triangle	
Quadrilateral	4	360	Square	
Pentagon			Regular pentagon	
Hexagon				
Heptagon			Regular heptagon	
	8			
Nonagon				
Decagon				
Undecagon				
Dodecagon				

8 Angles in a circle

8.1 Central angle

A central angle is an angle whose vertex is the center of a circle and whose sides pass through a pair of points on the circle.



There is an arc of the circumference associated to the angle and it is, by definition equal to the central angle itself.

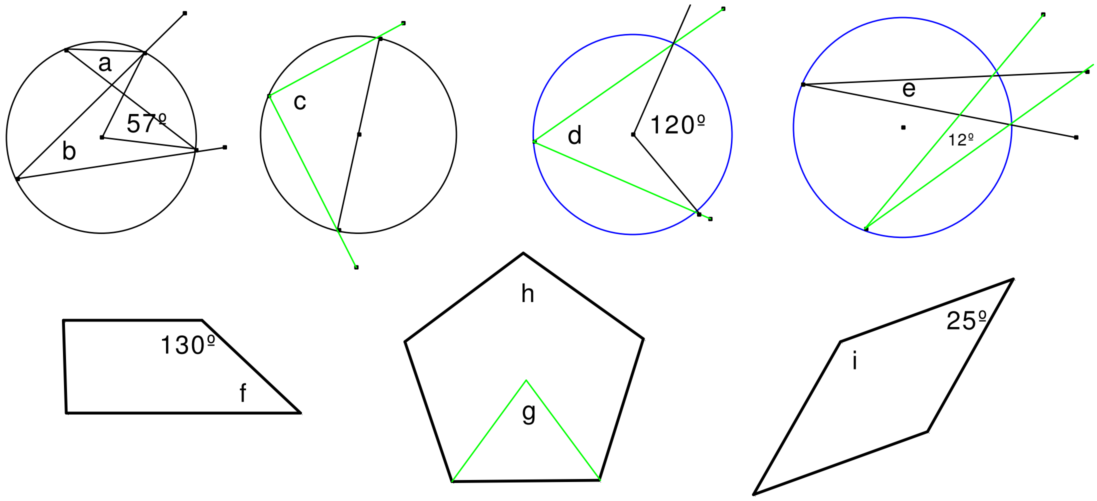
∠MON is a central angle of 60º

8.2 Inscribed angle

It is formed when two lines intersect a circle and its vertex is on the circumference. The measure of the intercepted arc in an inscribed angle is exactly the half of the central angle.

∠MPN is an inscribed angle with the same arc, so it is a 30º angle.

Exercise 10 Calculate the angles in each figure and explain your answer:



a = **b =**

c = **d =**

e = **f =**

g = **h =**

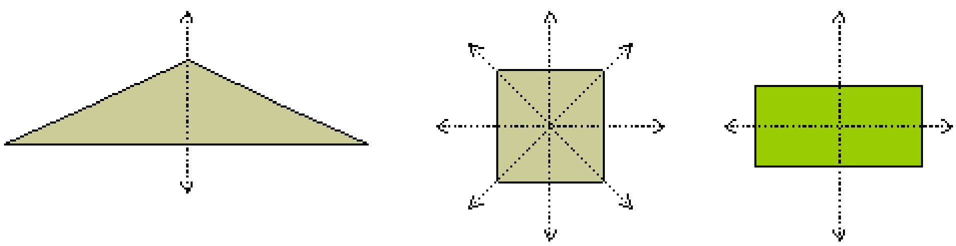
i =

9 Symmetric shapes

A figure that can be folded flat along a line so that the two halves match perfectly is a symmetric figure; such a line is called a line of symmetry.

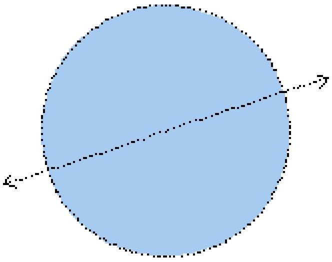
Examples:

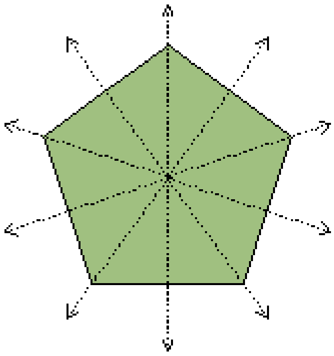
The triangle, the square and the rectangle below are symmetric figures. The dotted lines are the lines of symmetry.



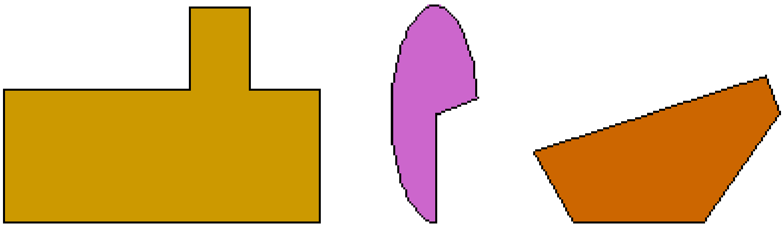
The regular pentagon below is a symmetric figure. It has five different lines of symmetry shown below.

The circle below is a symmetric figure. Any line that passes through its centre is a line of symmetry!



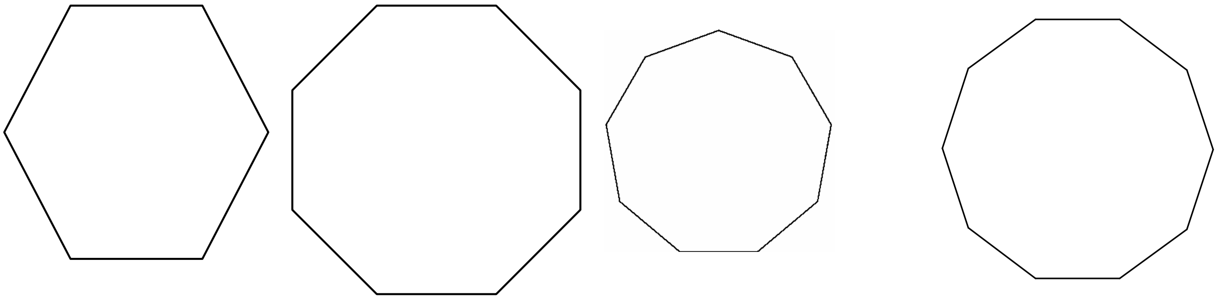


The figures shown below are not symmetric.



Exercise 11 Draw in all the lines of symmetry in the following regular polygons and use your answers to complete the table:

Polygon	Hexagon	Octagon	Nonagon	Decagon
Nº symmetry lines				



Solutions

Exercise 1 1.1 1.2 a) 45º, b) 104º, c) 22º, d) 113º, e) 71º. **Exercise 2** a) b) Left: obtuse $\angle BAC$, $\angle BDC$, $\angle EDF$, $\angle ACF$, $\angle ABE$; right: $\angle EBC$, $\angle BEF$, $\angle BCF$, $\angle CFE$; all the rest are acute angles. Right: acute: $\angle FEG$; obtuse: $\angle DFE$; all the rest are right angles. **Exercise 3** a) 90º, b) 180º, c) 30º, d) 120º, e) 6º, f) 276º. **Exercise 4** a) 73º, b) 48º, c) 83º, d) 0º. **Exercise 5** a) 143º, b) 128º , c) 57º, d) 173º, e) 90º. **Exercise 6 a)** a = 60º, b = 120º, c = 60º, d = 60º, e= 120º, f = 64º, g = 116º, h = 64º, i = 116º, j = 146º, k = 146º, l = 34º, m = 146º, n = 34º. **b)** 1. a = 148º, b = 148º, c = 32º, 2. a = 154º, b = 26º, c = 154º, 3. a = 80º, b = 60º, c = 80º, d = 120º, 4. a = 108º, b = 72º, c = 72º, d = 72º, e = 72º, f = 108º, 5. a = 80º, b = 60º, c = 40º, d = 40º, 6. a = 130º, b = 140º, c = 140º, d = 130º. **Exercise 7** 7.1 a) 186º 30’ 4’’, b) 61º 1’ 4’’, c)312º 54’ 39’’. 7.2 a) 23º 1’ 50’’, b) 76º 14’ 48’’, c) 86º 30’. 7.3 a) 45º 5’ 22’’, b) 67º 38’ 3’’, c) 4º 30’ 32’’, d) 9º 1’ 4’’. **Exercise 8**

Name	Number of sides	Sum of the interior angles	Name of the regular polygon	Interior angle
Triangle	3	180º	Equilateral triangle	60º
Quadrilateral	4	360º	Square	90º
Pentagon	5	540º	Regular pentagon	108º
Hexagon	6	720º	Regular hexagon	120º
Heptagon	7	900º	Regular heptagon	128º 36’
Octagon	8	1080º	Regular octagon	135º
Nonagon	9	1260º	Regular nonagon	140º
Decagon	10	1440º	Regular decagon	144º
Undecagon	11	1620º	Regular undecagon	147º 18’
Dodecagon	12	1800º	Regular dodecagon	150º

Exercise 10 a = 57º, b = 28º 30’, c = 90º, d = 60º, e = 12º, f = 50º, g = 72º, h = 108º, i = 155º. **Exercise 11**

Polygon	Hexagon	Octagon	Nonagon	Decagon
Nº symmetry lines	6	8	9	10